

NONLINEAR EXCHANGE RATE MODELING

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Our knowledge concerning exchange rates does not seem proportional to the effort. Despite being one of the most widely studied financial variables, very little is known about the behavior of the spot exchange rate. Foreign exchange rates seem to defy simple equilibrium relationships like interest parity.¹ Atheoretical models have fared no better. Once estimated, the data quickly expose models to be nothing more than curve fitting exercises.² Nonlinear modeling has emerged as a possible solution to this conundrum.

One branch of this new literature has adopted empirical models for the higher moments of exchange rates. Exchange rate returns are generally non-normal (in the Gaussian sense).³ Leptokurtic ("fat-tailed") distributions, with clustering of the large errors, has motivated the application of the generalized autoregressive conditional heteroscedasticity (GARCH) model of Engle (1982) and Bollerslev (1986) to exchange rates. The GARCH models have largely faltered on two grounds. A compelling theoretical explanation for the volatility clustering is needed.⁴ The GARCH effects seem also not to be present in the mean, and therefore are of little use for forecasting.

Nonlinear modeling has also been motivated by institutional changes. Since March of 1979, nearly all the major European currencies have been traded within target zones. Krugman (1991) has shown that target zones will introduce nonlinearities into exchange rates even if the bands never change.

¹ See e.g. Froot and Thaler (1990) and the resolution proposed in Mizrach (1993d).

² An exhaustive survey of models from the 1970s can be found in Meese and Rogoff (1983).

³ See e.g. Hsieh (1988).

⁴ See Mizrach (1990) for some work along these lines.

Several researchers have tried to make sense of the nonlinearities using time series techniques. Engel and Hamilton (1990) have used a Markov-switching model. Meese and Rose (1990, 1991), Diebold and Nason (1990), and Mizrach (1992) have used nonparametric techniques. None of these papers has produced robust out-of-sample forecast improvements. The random walk is often a better predictor than sophisticated nonlinear technology.

This paper tries to get a better understanding of the failures of the nonlinear models. I show that the spot exchange rate is indeed predictable most of the time. At a few crucial junctures, coinciding with exchange rate realignments, all models, linear and nonlinear, forecast poorly. I show that a new class of model introduced in Mizrach (1993b) can help us understand when forecasting is likely to be difficult. The Mizrach model produces a daily time series of the probability of realignment. Adopting a rule of thumb, to not forecast when the risk is more than two standard deviations above average, we can dramatically improve our overall accuracy.

The paper is organized as follows. I begin with the naive benchmark for the exchange rate, the unit root. I then turn to linear and nonlinear time series models, including Mizrach's probit-Markov model. Using the probit's risk estimates, I repeat the modeling exercise using the rule of thumb. A summary and conclusion follow.

1. The Random Walk

1.1 Data

For purposes of empirical illustration, I look at the French Franc, German Deutschemark (FF/DM) exchange rate over the period 13-March, 1979 to 11-September, 1992. This spans the creation the exchange rate mechanism (ERM) in Europe to the suspension of the Italian Lira and the British Pound from the ERM. I have 3417 daily observations on ecu exchange rates from the 14:30 fix in Basle which I convert into a cross rate.

ERM exchange rates float in 2.25% bands around a central parity. The central parity is occasionally realigned. With the Franc, the parity has been reset on seven occasions.⁵

1.2 Unit Root Tests

Let s_t denote the log the spot exchange rate, and let Δs_t be the log difference. I regress the difference on the lag of the level. A coefficient significantly less than zero would reject the null hypothesis of a unit root.

Table 1
Unit Root Tests

	13-Mar-79 to 11-Sep-92	# obs
Coeff.	-5.002E-04	3416
(t-stat)	(2.06)	

The relevant critical values are the Dickey-Fuller statistics, not the usual t-statistics.⁶ In a large sample, a 5% one-sided critical value is 2.86, and a 1% critical value is 3.43. Despite a t-ratio of 2.06, the coefficient is not significantly different from zero. This evidence supports the conventional wisdom that it is hard to reject the random walk as a statistical description for the spot exchange rate

1.3 Analysis of unconditional moments across regimes

I next looked at the unconditional moments of the first differences of the FF/DM exchange rate. The daily movements are quite small, but rates can still be quite volatile. The kurtosis for the sample as a whole is over 300.

⁵ On 4-August, 1993, the Franc passed below its old ERM floor of 3.4305 FF/DM. Bands of 15% have been introduced around the old parity, so technically, the Franc has not devalued.

⁶ For more discussion on this issue, see Mizrach (1993a). Mizrach bootstraps the critical values because of the non-normality and serial correlation in the data. He finds, in an experimental design a bit different from this one, that the appropriate critical values are 3.43 and 5.06.

Table 2
Descriptive Statistics

Δs_t	Full samp.	Omit reg ch.
Mean	11.457E-05	5.935E-05
SD	1.997E-03	1.253E-03
Kurt.	324.694	26.916

I wanted to examine whether the regime changes were influencing these results. I omitted 66 influential data points, five days around either side of a devaluation. These results appear in the second column of Table 2. The daily changes are much smaller because I have removed the large devaluations. Even more notable is that the kurtosis falls by a factor of more than 10, to 26.916.

2. Alternative Models for the Exchange Rate

I turn next to several different approaches to model the exchange rate. I then analyze the residuals to see which, if any, of the models does the best job of depicting the important devaluation episodes.

2.1 Linear time series

The most straightforward time series approach is a Box-Jenkins model. Using the Akaike criterion, I fit an AR(3) model to the first differences.

2.2 Near-neighbor models

Nonparametric approaches are appealing because they can provide meaningful statistical inference when very little is known about the series' fundamentals or distribution. Recent efforts at nonparametric modeling include Meese and Rose (1990,1991), Diebold and Nason (1990) and Mizrach (1992).

A technique that is amenable to our application is nearest neighbor methods. The idea is to find neighbors near to the current realization of the independent variables. With locally weighted

regression, one then fits a regression surface to the neighboring dependent variables. I selected a model with 5 neighbors and used least squares weights to estimate the exchange rate changes. I denote this model as 5-NN.

3. Analysis of the Residuals

In this section, I analyze the residuals of the two models to evaluate their performance in explaining the critical devaluation episodes.

The first column of Table 3 lists the dates of the 6 devaluation episodes. The next column is the percentage of the variance in the raw data due to the 66 realignment observations. Under the columns for the two models, AR(3) and 5-NN, are the percentages of the sum of squared residuals for the same dates.

Table 3
Analysis of Variance

Date	% Var	AR(3)	5-NN
9/17-10/1/79	0.37	0.39	0.20
9/24-10/8/81	13.81	14.35	13.93
6/7-6/21/82	25.24	25.16	24.79
3/15-3/30/83	12.54	8.73	12.59
3/24-4/10/86	8.63	8.81	8.49
1/5-1/19/87	0.83	0.83	0.73

Even though they comprise only 66 observations out of 3416, or less than 2% of the total, the six devaluation episodes explain 61% of the variance.

The AR(3) model does not explain these sudden devaluations. In the residuals of the AR(3) model, 58% of the sum of squared residuals is due to the 6 episodes.

The NN-model leaves just as much information behind in the residuals as do the linear AR models. 61% of the sum of squared residuals is in these devaluation episodes.

It seems that if we are to make much progress, we need to uncover something that helps us predict realignments.

4. A Probit-Markov Model of Devaluation Risk

In Mizrach (1993b), I introduce a new type of Markov-switching model. Unlike conventional switching models, the probability of a change in regime varies smoothly throughout the sample.

The model has two parts. In the first part, as with conventional switching models, one specifies models for the conditional mean in both regimes. In our exchange rate context, they are the within band and devaluation regimes. Within the band, the exchange rate is mean reverting. Outside of the band, I find that devaluations are proportional to the cumulative departure from purchasing power parity.

I will describe the probit part of the model in greater detail since that is what I will make use of in this section. I link the devaluation risk to two variables. The first is the position of the exchange rate within the band. Define

$$z_{2t} = (s_t - \underline{s}) / (\bar{s} - \underline{s}), \quad (1)$$

where $[\underline{s}, \bar{s}]$ are the lower and upper bounds of the target zone.

The second variable is based on the yield curve. During several devaluation crises, the term structure has become steeply negatively sloped. For example, on 15-March, 1983, 5 days prior to a realignment of the Franc, the French 3-month $i_t^{3/12}$, 1-month $i_t^{1/12}$ spread, denoted here as

$$z_{3t} = \log(1 + i_t^{3/12}) - \log(1 + i_t^{1/12}), \quad (2)$$

was -46.00. I also add a constant term, defining $z_t = (1, z_{2t}, z_{3t})$.

To ensure that the risk remain on $[0, 1]$, I make a probit transformation,

$$p_t = \int_{-\infty}^{\gamma z_t} (\sqrt{2\pi})^{-1} \exp(-t^2/2) dt \equiv \Phi(\gamma z_t). \quad (3)$$

In a fully specified model for the French-German interest differential, I obtain implicit market estimates of the potential devaluation risk. I find $\hat{\gamma} = (-1.563, 0.367, -17.177)$. I then compute a risk measure series, \hat{p}_t .

In Table 4, I look at the risk just prior to realignment. The first column contains the average risk in the 5 days prior to realignment. In the second column, I have the peak risk, which is almost always the day before the devaluation.

Table 4
Risk Estimates Prior to Realignment

Date	Avg. Risk	Peak Risk
24-Sep-79	8.56	9.11
05-Oct-81	32.11	37.80
14-Jun-82	35.31	43.70
21-Mar-83	99.99	99.99
07-Apr-86	19.67	25.91
12-Jan-87	14.72	19.75

These risks should be compared relative to a mean risk of devaluation of 8.3% with a standard deviation of 6.2%. In 5 of the 7 realignments, a risk two standard deviations above the mean (20.7%) was observed prior to a devaluation.⁷

Now we'll see whether this risk model can be useful in fitting the exchange rate data.

5. Model Evaluation

In regression exercises, I discovered that the risk model was not very precise in detecting the exact day of realignment. If you predicted a large change in the spot rate every day in which the risk was significantly above its mean, you would forecast very poorly. I chose instead to look at a rule of thumb where the risk measure provided information on when *not* to forecast.

In Table 5, I look at the sample mean squared errors (MSE) for the linear and nonparametric models. The random walk (a no change forecast) is included as a benchmark. Note that the MSE for the nonlinear 5-NN model is less than 0.2% better than the random walk (RW).

Table 5
Model Evaluation

Model	MSE	M-stat
Random Walk	4.01E-06	
5-NN	3.97E-06	0.03
AR(3)	3.95E-06	1.16
Risk AR(3)	1.37E-06	2.14

⁷ On 30-July, 1993, prior to the widening of the bands, the Franc's risk estimate was at 24.5%.

To make a formal comparison, I use the robust forecast comparison introduced in Mizraeh (1993c), which I designate the M-stat in the table. This statistic has very weak population assumptions which can readily handle the kurtosis we found in Section 1. It has an asymptotic normal distribution which is a good approximation in a sample of this size.

The last line of the table is a forecast rule of making no prediction when risk is more than two standard deviations above its mean (20.7% in our sample.) Using this rule of thumb, 56 observations are eliminated, but the MSE improves almost threefold. The M-stat shows that the improvement is statistically significant.

6. Conclusion

The idea that exchange rates are unpredictable needs to be qualified. The fixed exchange rates of the ERM are difficult to predict only at times of realignment. These regime changes, which contribute to the characteristic GARCH effects, are also predictable. We were able to improve our forecast accuracy almost 300% by limiting our predictions to those days in which the risk of realignment was not significantly higher than average.*

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